ScaffCC: A Framework for Compilation and Analysis of Quantum Computing Programs

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Quantum Computing Advantage

- A certain class of problems can be solved significantly faster by changing the paradigm of computing: use quantum mechanical systems to store and manipulate information.

- Example: Factoring a large $b$-bit number

<table>
<thead>
<tr>
<th></th>
<th>Asymptotic Complexity</th>
<th>232-digit number factoring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best classical algorithm (GNFS) [Buhler 1994]</td>
<td>$O(\exp(\frac{64}{9}b^{\frac{1}{3}}(\log b)^{\frac{2}{3}}))$</td>
<td>2000 years on a single-core AMD Opteron [Kelinjung et al. 2010]</td>
</tr>
<tr>
<td>Shor's quantum algorithm</td>
<td>$O(b^3)$</td>
<td>(technology dependent – theoretically large speedup)</td>
</tr>
</tbody>
</table>
Background on Quantum Computers

- A quantum bit *(qubit)* can exist in a *superposition* of states:
  \[ |\psi\rangle = \alpha |0\rangle + \beta |1\rangle \]
- Quantum operations *(gates)* transform the state of qubits.
- **Measurement** (observation) collapses it to either \(|0\rangle\) or \(|1\rangle\).
- Quantum computation is reversible.

```
Quantum Assembly
qbit a[1], b[5];
H(b[0]);
H(b[1]);
H(b[2]);
H(b[3]);
H(b[4]);
Z(a[0]);
CNOT(a[0], b[1]);
```
Compiling Quantum Codes

- Data types and instructions in quantum computers:
  - Qubits, quantum gates
- Decoherence requires QECC
  - Logical vs. Physical Levels
- Efficiency crucial
  - Inefficiencies at logical level are amplified into greater physical level QECC requirements.
Goals and Contributions

• 1) Identifying differences in compiling for quantum vs. classical computers
• 2) Providing good scalability to practical algorithm sizes
• 3) Automatically synthesizing reversible computation (e.g. for math functions)
• 4) Developing important program analysis passes
# Benchmarks

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Classical Time Complexity</th>
<th>Quantum Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grover’s Search</td>
<td>$O(n)$</td>
<td>$O(\sqrt{n})$</td>
</tr>
<tr>
<td>Binary Welded Tree (BWT)</td>
<td>$O(\frac{1}{4} 2^{\frac{n}{3}})$</td>
<td>$O(\frac{1}{4} k^4 n^9)$</td>
</tr>
<tr>
<td>Ground State Estimation (GSE)</td>
<td>$O(2^n)$</td>
<td>$O(n^5)$</td>
</tr>
<tr>
<td>Triangle Finding Problem (TFP)</td>
<td>$O(n^2)$</td>
<td>$O(n^{1.3})$</td>
</tr>
<tr>
<td>Boolean Formula (BF)</td>
<td>$O(n)$</td>
<td>$O(\sqrt{n})$</td>
</tr>
<tr>
<td>Class Number (CN)</td>
<td>$O((n \ln n)^{0.5})$</td>
<td>$O(\log(n) \log^* (n))$</td>
</tr>
</tbody>
</table>
# include <math.h>
#define n 5
#define N pow(2,n)

// module prototypes
module Sqr(qbit a[n], qbit b[n]);
module EQxMark(qbit b[n], qbit t[1], int tF);

// diffusion module
module diffuse(qbit q[n]) {
    // allocate qubits local to module
    qbit x[n-1];

    // Hadamard applied to q
    for(j = 0; j < n; j++)
        H(q[j]);
    ...
}

// main module
module main() {
    // allocated qubits in main
    qbit a[n], b[n], t[1];

    // classical bits : measurement outcome
cbit ma[n];

    // iteration bound
    int nstep = floor((pi/4)*sqrt(N))
    .
    .

    // Grover iteration: Repeat O(N^0.5) times
    for (istep=1; istep<=nstep; istep++) {
        Sqr(a, b);
        EQxMark(b, t, 0);
        Sqr(a, b);
        diffuse(a);
    }

    // measure a to find outcome
    for(i=0; i<n; i++)
        ma[i] = measZ(a[i]);
From Scaffold to QASM: Deep Optimization through LLVM

- ScaffCC translates from **Scaffold** Programming Language to **QASM** assembly language.
  - Implemented with **LLVM**, a rich and mature compiler framework.
  - Modified **Clang** front-end parses and converts ScaffCC to LLVM Intermediate Representation.
Scalability in Compilation and Analysis (1)

• Quantum circuits are typically specialized to one problem size, hence they are deeply and statically analyzable.
  – Classical control resolution

• Static classical control resolution using LLVM passes
  – May cause code explosion during code transformation of larger problems
Resolving Classical Controls in the Code

• Classical control surrounding quantum code must be resolved to disambiguate for the hardware the qubits and the exact set of gates

```c
#define s_ 3000
module Oracle(qbit a[1], int j) {
    double theta=(-1)*pow(2,j)/100;
    Rz(a[0],theta)
}
module main() {
    qbit a[1];
    int i,j;
    for (i=0;i<=s_;i++)
        for (j=0;j<=3;j++)
            Oracle(a,j);
}

module Oracle_0(qbit a[1]) {
    Rz(a[0],-0.01);
}
module Oracle_3(qbit a[1]) {
    Rz(a[0],-0.08);
}
```
module EQxMark (qbit b[n], qbit t[1], int tF) {
    .
    if(tF==1)
        CNOT(t[0], x[n-2]);
    else
        Z(x[n-2]);
    .
}

module EQxMark_0 (qbit b[n], qbit t[1]) {
    .
    Z(x[n-2]);
    .
}

module EQxMark_1 (qbit b[n], qbit t[1]) {
    .
    CNOT(t[0], x[n-2]);
    .
}

module main (qbit b[n], qbit t[1]) {
    .
    for (i=0; i<2; i++)
        EQxMark(b, t, i);
    .
}

module main (qbit b[n], qbit t[1]) {
    .
    EQxMark(b, t, 0);  EQxMark_0(b, t);
    EQxMark(b, t, 1);  EQxMark_1(b, t);
    .
}
Pass-Driven Vs. Instrumentation-Driven

• **Pass-Driven:**
  – Loop unrolling
  – Procedure Cloning
  – Inter-procedural Constant Propagation

• **Instrumentation-Driven:**
  – Leveraging the dual nature of quantum programs
  – Instrument code such that a fast classical processor executes through the classical portion, collecting information regarding the quantum portion
  – Further speed-up by memoizing same module calls
The Instrumentation-Driven Approach Scales Better
Scalability in Compilation and Analysis (2)

• Traditional QASM:
  – No loops or modules: only sequences of qubits and gates
  – Used for small program representations

• Programs that we examined contained between $10^7$ to $10^{12}$ gates

• We need a more scalable output format:
  – QASM with Hierarchy (QASM-H)
    • 200,000X smaller code
  – QASM with Hierarchy and Loops (QASM-HL)
Managing Scalability with QASM Format

**Scaffold**

```c
#define n 1000
module foo(qbit q[n]) {
    for(int i = 0; i < n; i++)
        H(q[i]);
    CNOT(q[n-1],q[0]);
}
module main() {
    qbit b[n];
    foo(b);
}
```

**QASM-H**

```qasm
module foo(qbit* q) {
    H(q[0]);
    H(q[1]);
    ...
    H(q[999]);
    CNOT(q[999],q[0]);
}
module main() {
    qbit b[1000];
    foo(b);
}
```

**Flat QASM**

```qasm
qbit b[1000];
H(b[0]);
H(b[1]);
...
H(b[999]);
CNOT(b[999],b[0]);
```

**QASM-HL**

```qasm
module foo(qbit* q) {
    H(q[0:999]);
    CNOT(q[999],q[0]);
}
module main() {
    qbit b[1000];
    foo(b);
}
Comparison of QASM-H and QASM-HL

- A large reduction is already obtained from QASM-H over flat QASM.
Synthesizing Reversible Computation

- Classical-To-Quantum-Gate (CTQG): A ScaffCC feature for efficiently translating classical modules to quantum modules.
CTQG: Classical-To-Quantum-Gate

• Facilitates the synthesis of quantum circuits from classical mathematical expressions:
  – Basic **integer** arithmetic (a=a+b, a=a+bc, ...)
  – **Fixed-point** arithmetic (1/x, sin x, ...)
  – **Bit-wise** manipulations (shift operators, ...)

• State-of-the-art in reversible logic synthesis, minimizing the use of extra (**ancilla**) qubits

• Produces output gate-by-gate on the fly
  – Not limited by memory
Program Analysis

- Analysis passes:
  - Program correctness checks
  - Program estimates
Program Analysis

• ScaffCC supports a range of code analysis techniques:
  – Program correctness checks:
    • No-cloning checks
    • Entanglement and un-computation checks
  – Program estimates:
    • Resource estimation
    • Timing analysis (Parallel scheduling)
Program Correctness Checks

• No-Cloning:
  – Theorem: The state of one qubit cannot be copied into another (no fan-out)
  – Check that multi-qubit gates do not share qubits

• Entanglement:
  – The joint state of two qubits cannot be separated
  – Data-flow analyses to automate the tracking of entanglement and disentanglement
Quantum Program Analysis: Resource Analysis

• Obtaining estimates for the size of the circuit:
  – Qubits are expensive
  – More gates require more overall error correction and hence more cost

• The same pass-driven and instrumentation-driven approaches apply

• Dynamic memoization table records number of resources

<table>
<thead>
<tr>
<th>Module</th>
<th>IntegerParam</th>
<th>DoubleParam</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Qubit</td>
</tr>
<tr>
<td>main</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Oracle</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oracle</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oracle</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oracle</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Timing Estimate

- Estimates the critical path length of the program
  - Assuming unlimited hardware capability for parallelization
- Scheduling based on qubit data dependencies between operations
- Hierarchical scheduling for tractability:
  - Obtain module critical paths separately and then treat them as black boxes.
Remodularization

• Analysis makes use of modularity
  – Avoid repetitive analysis
  – Reduce analysis time
• Results in loss of parallelism at module boundaries
  – Decreased schedule optimality
• Idea:
  – Inline small modules at call sites – larger flattened modules
  – Define threshold for “small” modules
  – Results in better critical path estimates
Hierarchical Approach Tradeoff

• Closeness to actual critical path is dependent on the level of **modularity**
• Flatter overall program means more opportunity for discovering parallelism
Effect of Remodularization

• Based on resource analysis, flatten modules with size less than a threshold
• Tradeoff between speed of analysis and its accuracy
Conclusion

• Extended LLVM’s classical framework for quantum compilation at the logical level
• Managed scalability through:
  – Output format:
    • 200,000X on average + up to 90% for some benchmarks
  – Code generation approach:
    • Up to %70 for large problems
• CTQG: Automatic generation of efficient quantum programs from classical descriptions
• Developed a scalable program analysis toolbox
• ScaffCC can be used as a future research tool